

List of Review Topics



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Geometry

- 1) Describe the Pythagorean theorem.
- 2) Describe the equivalent of Pythagorean's theorem in a 3-D space? (De Gua's Theorem)
- 3) Let r be the inradius of a $\text{Rt}\triangle ABC$ whose sides are a , b and c , respectively, where c is the hypotenuse. Express r using a , b , and c .
- 4) Let CD be the altitude drawn to the hypotenuse of a $\text{Rt}\triangle ABC$. If r_1 , r_2 and r_3 are the inradii of $\triangle ACD$, $\triangle BCD$ and $\triangle ABC$. Show that $CD = r_1 + r_2 + r_3$.
- 5) Three circles are tangent to each other and also to a common line on the same side. If their radii are r_1 , r_2 and r_3 , respectively, where r_3 is the smallest, show $\frac{1}{\sqrt{r_3}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$.
- 6) Given two segments AB and MN , show that $AB \perp MN \Leftrightarrow AM^2 - BM^2 = AN^2 - BN^2$.
- 7) What is the Pythagorean triplet's formula? (also appear in Indeterminate Equation).
- 8) Describe the angle bisector theorem.
- 9) Can the angle bisector theorem be applied to exterior angle?
- 10) What is Stewart theorem?
- 11) (Apollonius' Theorem) Let the sides of $\triangle ABC$ be a , b , and c , respectively. Find the length of its median AM .
- 12) Name two sufficient and necessary conditions for a quadrilateral $ABCD$ to be inscribed to a circle.
- 13) What is the Ptolemy theorem?
- 14) What is the power of point theorem?
- 15) What is the Ceva's theorem?
- 16) How to prove the three medians of any given triangle are concurrent?
- 17) How to prove the three interior angle bisectors are concurrent?
- 18) What is the Menelaus' theorem?
- 19) Will Menelaus' theorem still hold if the transversal does not intersect any of the triangle's sides?
- 20) Let R , r and S be $\triangle ABC$'s circumradius, inradius and area, respectively. What are the relationship between R and S , as well as r and S ?
- 21) Let $\angle BAD = \alpha$ and $\angle CAD = \beta$. Show that point D is on BC if and only if $\frac{\sin(\alpha+\beta)}{\sin \alpha \sin \beta} = \frac{\sin \alpha}{\sin \beta} + \frac{\sin \beta}{\sin \alpha}$.
- 22) What is the Heron's formula?
- 23) Let a , b and c be the side lengths of $\triangle ABC$ and r be its inradius, express r using a , b , and c .
- 24) How to extend Heron's formula to an inscribed quadrilateral?
- 25) If AB and PQ , or their extensions, intersect at point M , then $S_{\triangle PAB} : S_{\triangle QAB} = PM : QM$.
- 26) What are the Law of Sines and Law of Cosines?
- 27) What is the Law of Tangents?

List of Review Topics



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- 28) If the lengths of all the four sides of an inscribed quadrilateral are known, how to compute the lengths of its diagonals?
- 29) What is the relationship among the side lengths of a circumscribed quadrilateral?
- 30) Describe Ceva's theorem in trigonometric form.
- 31) What are some commonly used properties of a triangle's centroid?
- 32) How to use center of mass to solve some area related problems?
- 33) What orthocenter is often associated with concyclic points?
- 34) What is the Fermat point?
- 35) What is Euler's line?
- 36) What is the nine-point circle?

Coordinate Geometry

- 1) Given two points (x_1, y_1, z_1) and (x_2, y_2, z_2) , what is their distance?
- 2) If a plane's intercepts of x , y , and z axes are a , b , and c , respectively, what is this plane's equation?
- 3) What are the conditions for two lines $y = k_1x + m_1$ and $y = k_2x + m_2$ to be (1) parallel, (2) perpendicular?
- 4) If $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ represents the same line, what conclusion can we draw?
- 5) What is the distance between point (x, y) to the line $Ax + By + C = 0$?
- 6) What is the distance from the origin to the plane $Ax + By + Cz + D = 0$?
- 7) If a plane intersects x , y , and z axes at points A , B , and C , respectively, what is the area of $\triangle ABC$? Try to find two methods.
- 8) Let the coordinates of points A and B be (x_a, y_a) and (x_b, y_b) . If point P lie on segment AB where $\overline{AP} : \overline{PB} = m : n$. Find point P 's coordinates. What if P is on AB 's extension?
- 9) Given three points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) . What is the area of the triangle formed by these three points?
- 10) Let point P be on circle $x^2 + y^2 = r^2$. Find the equation for the line which is tangent to the circle at P .

Algebra

- 1) How to factorize symmetric polynomial?
- 2) What is the Vieta's formula in the case of
 - i) a quadratic equation?
 - ii) for an equation of n degree?
- 3) Describe three different ways to evaluate $x_1^n + x_2^n$ where x_1 and x_2 are roots of a quadratic equation.
- 4) What are some typical root related problems and their methods?
- 5) What is the usual ways to solve a high degree equation?
- 6) How to solve function equations?

List of Review Topics



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Combinatorics

- 1) What are the three basic counting principles?
- 2) How to count the number of divisors for any given positive integer?
- 3) How to handle counting problems which involves round-table / circle?
- 4) What is a geometric probability question and how to solve it?
- 5) What is the φ function?
- 6) What is the lattice method? What are some typical problems that can be solved by using this method?
- 7) What is the bundling method? Give a typical problems that can be solved by using this method.
- 8) What is the symmetric method? Give a typical problems that can be solved by using this method.
- 9) Given $x_1 + x_2 + \cdots + x_k = n$ where k and n are positive integers, find
 - i) number of positive integer solutions.
 - ii) number of non-negative integer solutions.
- 10) What if the restrictions of x_i are different in $x_1 + x_2 + \cdots + x_k = n$? For example, some are positive integers, some are non-negative, and some are within a specific range?
- 11) How to count the different possibilities to to put n balls in k baskets if
 - i) Both balls and baskets are distinguishable
 - ii) Neither is distinguishable
 - iii) One of them is distinguishable

Is there any difference if empty basket is or is not allowed?
- 12) What is the hockey stick identity?
- 13) How to compute the sum of all the coefficients in the expanded form of $(1 + x)^n$.
- 14) What are independent events and mutually exclusive events?
- 15) What is the tree method for solving probability related problems?
- 16) What is a generating function? How to use it to solve counting problems?

Trigonometry

- 1) What are the values of $\sin 15^\circ$ and $\cos 15^\circ$?
- 2) What are the value of $\sin 18^\circ$?
- 3) What are the sum of angles' formulas?
- 4) If $\tan \beta = \frac{1+\tan \alpha}{1-\tan \alpha}$, what is the relationship between α and β ?
- 5) What are the double angle formulas?
- 6) What are the half-angle formulas?
- 7) What is the usual approach to reduce square of a trigonometry function to a linear expression? i.e. 2^{nd} power to 1^{st} .
- 8) What are the sum-to-product and product-to-sum formulas?
- 9) Show that $\sin^2 \alpha - \sin^2 \beta = \sin(\alpha + \beta) \sin(\alpha - \beta)$ and $\cos^2 \alpha - \cos^2 \beta = -\sin(\alpha + \beta) \sin(\alpha - \beta)$
- 10) Describe the two forms of triple angle formulas and prove them.

List of Review Topics



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Complex Number

- 1) What do the following conditions mean?
 - i) $z = \bar{z}$
 - ii) $z \cdot \bar{z} = 1$
- 2) What is the De Moivre's formula?
- 3) What is Euler's formula in complex number?
- 4) What is the value of i^i ?
- 5) How to use Vector operations to represent rotating an angle?
- 6) How to use complex number to evaluate trigonometric expressions?
- 7) Use the complex number method to prove the trigonometric triple angle formula.

Indeterminate Equation

- 1) Describe the following methods.
 - The factorization method
 - The difference of square method
 - Sum of squares
 - The inequality method
 - The quadratic method.
- 2) How to solve $x + xy + y = n$?
- 3) How to solve $ax + by = c$?
- 4) How many positive integer solutions exist to the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ where n is a positive integer?
- 5) How many positive integer solutions exist to the equation $\frac{1}{x} + \frac{1}{y} = \frac{m}{n}$ where n and m are two positive integers?
- 6) How to solve the indeterminate equation $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{n}{m}$ where both n and m are positive integers?
- 7) How to solve $xy + yz + zx = kxyz$ where k is a positive integer?
- 8) What is the MOD method?
- 9) How to solve $ax + by = 1$?
- 10) Describe the Pythagorean triplet formula.
- 11) What is the infinite descent method?
- 12) What is a Pell's equation, and how to solve it?

List of Review Topics



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Sequence

- 1) How to find the sum and the n^{th} element of an arithmetic sequence?
- 2) How to find the sum and the n^{th} element of a geometric sequence?
- 3) Under which condition an infinite geometric sequence will have a sum? And what is its sum formula?
- 4) What is the typical method to solve sequences problems in AMC/AIME level competition? Given a few examples.
- 5) Give an example which can be solved by applying the relation $a_n = S_n - S_{n-1}$.
- 6) Explain why the sum of n consecutive terms in an arithmetic sequence must be in the form of $An^2 + Bn$ where A and B are some real numbers.
- 7) How to solve sequence in the form of $a_n = pa_{n-1} + q$ where p and q are two constants.
- 8) How to solve an sequence of $a_n = pa_{n-1} + qa_{n-2}$ where p and q are two constants.

Number Theory

- 1) What are the basic rules for modular arithmetic operations, such as addition, subtraction, multiplication, division, and exponen
- 2) What is the negative one technique?
- 3) What is the continuous simplification technique?
- 4) What is Euler's theorem?
- 5) What is the positive one technique?
- 6) What is modular inverse?
- 7) How to solve the ending digit problem? Unit only? Last two digits?
- 8) What is the divide-by-nine technique? Give a few typical problems that can be solved by this technique.
- 9) $x^2 + y^2 \not\equiv 3 \pmod{4}$
- 10) What is the most elementary approach to solve a modular equation?
- 11) Chinese Remainder Theorem (CRT)
- 12) Multiplicative order.
- 13) What is the Bezout identity?

Power Calculation

- 1) How to compute $1 \times 2 + 2 \times 3 + \cdots + n \times (n + 1)$?
- 2) How to compute $1 \times 2 \times 3 + 2 \times 3 \times 4 + \cdots + n \times (n + 1) \times (n + 2)$?
- 3) How to compute $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{n \times (n+1)}$?
- 4) How to compute $\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \cdots + \frac{1}{n \times (n+1) \times (n+1)}$?
- 5) How to simplify $\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$?
- 6) How to simplify $\sqrt{2 + \sqrt{3}}$?

List of Review Topics



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- 7) How to compute $C_n^0 + C_n^1 + \cdots + C_n^n$?
- 8) How to compute $C_n^0 + C_n^2 + C_n^4 \cdots$?
- 9) How to compute $C_n^0 + C_n^3 + C_n^6 + \cdots + C_n^n$?
- 10) How to compute $C_n^0 + \frac{1}{2}C_n^1 + \frac{1}{3}C_n^2 + \cdots + \frac{1}{n+1}C_n^n$?